# Supersymmetry breaking in ISS coupled to gravity

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ABSTRACT: We analyse the breakdown of supersymmetry in an ISS model in the presence of gravity, under the requirement that the cosmological constant vanishes dynamically. The gravitational backreaction is calculated in the metastable minimum and, in conjuction with the condition $V = 0$, this is shown to generate non-zero F-terms for the squarks. Once the squarks are coupled to the messenger sector, a gauge mediation scheme is realised and it leads to a distinctive soft spectrum, with a two order of magnitude split between the gaugino and the soft scalar masses.

KEYWORDS: Supersymmetry Breaking, Supergravity Models, Supersymmetry and Duality

## Contents

## 1   Introduction

In this letter we analyse the meta-stable point of a simple Intriligator, Seiberg and Shih (ISS) [1] model, within the framework of supergravity. This allows us to cancel the cosmological constant, which we opt to do by the simplest possible method: adding a constant, $W_0$, to the superpotential. This is sufficient to generate a physically reasonable gravitino mass and balance the new negative contribution to the potential against the original positive contribution coming from the ISS potential.

We recompute the one-loop effective potential in supergravity and use this to compute the gravitational backreaction on the global vacuum.[1] The perturbations are shown to be small, as one expects from gravitational corrections, but non-trivial. We stress that it is necessary to consider gravitational corrections, even though we know they are small. They are important when determining the expectation values of the fields, most notably

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[1]For an earlier study with somewhat different findings, see [2]. An interesting study that coupled a realistic moduli sector to an ISS model can be found in [3].

the moduli, but the remaining fields are shifted more than dimesional analysis would suggest. The most interesting effect is the generation of non-zero, but Planck suppressed, F-terms for the magnetic quarks. Hence, there appear two distinct scales in the sector that breaks supersymmetry.

It is interesting to calculate the relative importance of several mediation mechanisms in this setup, specifically anomaly, gravity and gauge mediation. We give order of magnitude estimates for the soft masses spectrum generated by these three mechanisms and argue that the spectrum can have a striking gap between the gaugino masses and the soft scalar masses. This is reminiscent of split SUSY [4], but the split is not allowed to be arbitrarily large since it is constrained by the requirement that $V = 0$ in the meta-stable minimum.

We note that setting $V = 0$ at tree-level is clearly not sufficient to guarantee it remains close to zero when loop corrections are included. As discussed in detail in [5, 6] one generically expects both the logarithmic corrections present in the SUSY theory (albeit with gravitationally corrected masses) and quadratically divergent contributions, $V_{\text{quad.}} = \frac{1}{32\pi^2}\text{STr}\mathcal{M}^2\Lambda^2$, to be present if the theory is cut-off at $\Lambda$. However, as noted in [6] and discussed further in [7], this contribution is determined by the geometry of the Kähler potential and the number of degrees of freedom in the effective theory, and in principle it is possible for it to vanish. Even if it remains, its presence is not necessarily particularly damaging, since it is fixed by the size of $m_{3/2}$.

The potential can then be parametrised as (with $M_P$ set to 1 and $V_{\text{log}}$ denoting the logarithmic one-loop contribution):

$$V = V_F + V_{\text{log}} + (Z - 3)m_{3/2}^2 \tag{1.1}$$

where $Z$ is a parameter encapsulating our ignorance about UV effects and is $\mathcal{O}(\frac{1}{32\pi^2})$ to $\mathcal{O}(\frac{N_{\text{TOT}}}{32\pi^2})$, where $N_{\text{TOT}}$ is the total number of chiral fields. If $Z < 0$, the condition $V = 0$ is satisfied by a smaller $W_0$ than is required to cancel the tree-level potential. Since we know that $|Z| \propto \Lambda^2$ this implies that it must be possible to chose a cut-off small enough that $W_0$ will not change dramatically, and our results will be qualitatively unchanged with respect to the case with the quadratically divergent term omitted. We have assumed, and prove in appendix B, that the derivatives of $V_{\text{quad.}}$ are similar in form and magnitude to $V_{\text{tree}}$'s. It is interesting to note that $V_{\text{quad.}}$ can play an important role in the potential despite being generated by gravity, in close analogy to the role played by $-3e^K|W|^2$. This is in contrast to the gravitational corrections to the logarithmic potential, which are negligible in comparison to the globally supersymmetric terms. Naturally, we still have to re-tune to get $V = 0$, but the loop corrections do not increase the degree of tuning required. Finally, if $Z \gtrsim 3$ it is clear that these models break down and the cosmological constant cannot be tuned to zero. This will not be the case unless the cut-off is close to the Planck scale. In these models, the relevant cut-off is the scale at which the magnetic description is no longer valid as supersymmetry is best described by the low energy variables in the magnetic theory.

We have implicitly assumed, in using $N = 1$ supergravity formalism that the UV preserves one supersymmetry. On top of this, for simplicity's sake, we assume that the sole source of SUSY breaking is the ISS sector, with the constant $W_0$ setting the scale of $m_{3/2}$,

essentially postulating that $W = W_{\text{ISS}} + W_{\text{UV}}$, $\langle W_{\text{UV}} \rangle = W_0 \neq 0$, that $F_{\text{UV}} \ll F_\Phi$ and that the UV has been decoupled. While the constant can be dynamically generated in a explicit model we do not attempt to do so here (for an example where a KKLT model [8] is used in the UV see [9]). Finally we note that the UV's contributions to the Kähler geometry, and hence $V_{\text{quad.}}$, are uncalculable, but should be small on dimensional grounds.

## 2 Global ISS review

ISS showed that meta-stable SUSY breaking is possible in a wide class of remarkably simple models. One of their main examples is supersymmetric QCD with $N_f$ flavours and $N_c$ colours. If one lies in the free magnetic range, $N_c < N_f < \frac{3}{2} N_c$, then the low energy theory is strongly coupled, but admits a dual interpretation in terms of IR-free, magnetic variables.

The tree-level potential in the magnetic theory is given by an R-symmetry preserving O'Raifeartaigh model [10] and so SUSY has to be spontaneously broken: $F_i = 0$ cannot be satisfied for all fields.

The tree-level superpotential in the magnetic theory is given by:

$$W_{\text{tree}} = h \text{Tr}\big(\phi \Phi \tilde{\phi}\big) - h\mu^2 \text{Tr}(\Phi) \tag{2.1}$$

where $\Phi$ transforms as $N_f \times \overline{N_f}$, $\phi$: $(\overline{N_f}, N)$, $\tilde{\phi}$: $(N_f, \overline{N})$, $N = N_f - N_c$, the number of squark flavours in the magnetic theory and we denote the parts of $\Phi$ that will later obtain expectation values as follows: $\Phi = \begin{pmatrix} \Phi_1 & 0 \\ 0 & \Phi_0 \end{pmatrix}$. The Kähler potential is canonical.

Considering the tree-level superpotential in isolation one finds that the lowest energy state is a moduli space parametrised by

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix}, \qquad \phi = \begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}, \qquad \tilde{\phi}^T = \begin{pmatrix} \tilde{\phi}_0 \\ 0 \end{pmatrix}, \qquad \phi_0 \tilde{\phi}_0 = \mu^2 \, \mathbb{I}_{N_c \times N_c}. \tag{2.2}$$

Since SUSY has to be broken, the potential is positive definite and is found to have an expectation value of $V = h^2 N_c \mu^4$. When the one-loop effects are included the moduli space is lifted and, aside from flat directions identified with Goldstone bosons, a unique minimum is found at:

$$\Phi = 0, \qquad \phi_0 = \tilde{\phi}_0 = \mu \, \mathbb{I}_{N_c \times N_c}. \tag{2.3}$$

In addition one must include the non-perturbative, R-symmetry violating contribution:

$$W = N h^{N_f/N} \big(\Lambda_m^{-(N_f - 3N)} \det(\Phi)\big)^{1/N}. \tag{2.4}$$

Notice that the exponent of $\Lambda_m$, $-(N_f - 3N) = -(3N_c - 2N_f)$, is always negative in the free magnetic range. Hence the coefficient of the determinant grows as the cut-off shrinks.

Since the non-perturbative piece is R-symmetry violating a SUSY preserving minimum must exist [11], created by the non-perturbative piece. In global SUSY[2] this must be at a lower energy than the SUSY breaking minimum.

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[2] The situation could be improved in Sugra if the SUSY preserving point also had $W = 0$ and the SUSY breaking point $V = 0$, but this is difficult to obtain, and not the case here. In-fact, the difference in the energy density is increased by the negative contributions from $W \neq 0$.

## 2.1 A note on dynamical scales

We now calculate the relationship between the dynamical scales, $\Lambda$ and $\Lambda_m$, of the electric and magnetic theories, respectively. We make use of the relevant part of the dictionary given in ISS's paper and the duality relation, given by:

$$h = \frac{\sqrt{\alpha}\Lambda}{\widehat{\Lambda}} \tag{2.5}$$

and

$$\Lambda^{3N_c-N_f}\Lambda_m^{2N_f-3N_c} = (-1)^{N_f-N_c}\widehat{\Lambda}^{N_f}. \tag{2.6}$$

where $\hat{\Lambda}$ is a dimensional parameter in the magnetic theory, related to the electric quark mass, $m_0$, and the magnetic quark mass, $\mu$ through the following relation: $\hat{\Lambda} = -\frac{\mu^2}{m_0}$

If we assume that the order one number, $\alpha$, appearing in the Kähler potential for the electric mesons[3] is simply 1 and that $h = 1$, then it follows that all three scales, $\Lambda$, $\Lambda_m$ and $\hat{\Lambda}$ are identified, up to flavour dependent phases. Above this scale the electric description is valid, while the magnetic description is valid below.

## 3 Locally supersymmetric ISS

If one simply promotes ISS to having a local supersymmetry without including any additional physics, the results are not significantly perturbed near the minima of the SUSY theory.

However, the picture changes if any other terms appear in the superpotential. Any new physics that generates a non-zero $\langle W \rangle$, necessary to have a finite gravitino mass and cancel the cosmological constant, will at least interact gravitationally with the moduli.

Even the simplest possible modification, the addition of a small ($\ll 1$) constant, $W_0$, to the superpotential, is sufficient to push the pseudo-moduli to large expectation values. This is not altogether surprising since the global, tree-level potential is independent of the pseudo-moduli and so their entire potential is given by Planck suppressed, non-renormalisable operators once supergravity corrections are included. As such, the natural scale for their expectation values is $M_p$.

However, one-loop effects should not be ignored. In the vicinity of the metastable point the logarithmic one-loop potential generates mass corrections of order $h^2\mu$ multiplied by a loop suppression factor. For comparison, the typical contribution to the logarithmic part of the one-loop potential from the gravitational effects is $(h^2\mu^3/M_P)^{1/2}$.[4] Hence, the gravitational corrections to the logarithmic one-loop potential, while non-zero, will be small. This does not hold for the quadratic corrections which give mass corrections of order $h\mu$, suppressed by the cut-off and a loop factor.

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[3]$K_M = \frac{1}{\alpha|\Lambda|^2}\mathrm{Tr}M^{\dagger}M$.

[4]This can be derived assuming that $W_0 \sim \mu^2$ which will be required for cancellation of the cosmological constant; $hK_iW_0F$ gives a contribution to the mass square matrix of order $h^2K_iW_0$, i.e. $h^2\mu^3$.

In the following section we will consider the following simplified model, with the non-perturbative piece removed. This will allow us to isolate the effects of the constant, $W_0$, appearing as follows:

$$W = h\text{Tr}\left(\phi\Phi\tilde{\phi}\right) - h\mu^2\text{Tr}(\Phi) + hW_0 \tag{3.1}$$

The presence of this constant slightly changes the global SUSY minimum, introducing a modest amount of SUSY breaking. One can tune the constant such that the superpotential vanishes with the F-terms, but it is not possible to achieve this if we wish to have $V = 0$ at the metastable point.

The constant creates an AdS minimum with a negative expectation value equal in magnitude to the global ISS theory's, namely $V_{\text{ADS}} \simeq -h^2 N_c \mu^4$. However, the difference between $V$ in the AdS minimum and in the metastable minimum is essentially the same as the the difference between $V$ in the SUSY minimum and in the metastable in the global case. The height of the barrier is also essentially the same in both cases.

Finally our numerical studies demonstrate that if $W_0 \sim \mu^2$ then $\Phi_0$ gets expectation values of order 1, but the expectation value shrinks as $W_0 \to 0$, going to zero in that limit.

## 4  One loop potential

Since the interplay between the supergravity and one-loop effects is so important to our results it is worth discussing the details of the one-loop calculation, highlighting the approximations we have made. First of all we note that the mass matrices, $\mathcal{M}$, in the well-known formula:

$$V_{\text{one-loop}} = V_{\text{quad.}} + V_{\text{log}} = \frac{1}{32\pi^2}\text{STr}\mathcal{M}^2\Lambda^2 + \frac{1}{64\pi^2}\text{STr}\mathcal{M}^4 \log\frac{\mathcal{M}^2}{\Lambda^2} \tag{4.1}$$

are given by the supergravity corrected masses [12, 13]. This modifies the mass squared matrices at the level of $\mu^4$ (i.e. a rather small shift, but calculable and necessary for the computation of $V_{\text{quad.}}$). The term quadratic in $\mathcal{M}$ is generically $\sim m_{3/2}^2\Lambda^2$ whereas the contribution from $V_{\text{tree}}$ is $-3m_{3/2}^2$, hence the $\mathcal{M}^2$ term can be disregarded if $\Lambda \ll M_P$, but not otherwise. Unfortunately eq. (4.1) is modified when $V \neq 0$ (for discussions of this point, see [5] and [14]). Even though we are expanding about $V = 0$, there will be corrections to this expression since $V = 0$ is only true at that point and in the Goldstone directions. We expect there to be both logarithmic and quadratic corrections stemming from this. The quadratic terms we can ignore if $\Lambda \ll 1$, but the logarithmic terms we have to consider more carefully. We note that all the operators in [14] that contain $V$ are dimension 8 and so the largest possible linear contribution would be $\mathcal{O}(W_0^2\mu X) \sim \mu^5 X$ where $X$ is a generic field. This should be compared to the largest contribution from gravity at tree-level, $XFW_0 \sim \mu^4 X$, and so we expect these effects to be negligible.

As noted in [1] and [15] one can capture some information about the effective potential purely by integrating out fields and calculating the correction to the Kähler potential. However, as described in the appendix of [1], this is an approximation only valid to 2nd order in $F$, we also note that it is harder to work with numerically. Hence we opt to calculate the full one-loop potential. It is nonetheless interesting to compare these two

approaches and we see that the (somewhat arbitrary) corrections to the Kähler potential introduced in [16], created an explicit cut-off dependence into the effective potential for $\Phi_0$ and hence $\langle \Phi_0 \rangle \propto \Lambda^2$. This dependence is not present in global theory and we found that only a very mild dependence was introduced by including supergravity corrections to the logarithmic effective potential, as we demonstrate in section 6.1. Regrettably, this does not provide a rigorous test of the two approaches, due to the Kähler corrections being more postulated than derived in [16].

It is then not entirely surprising that our results differ markedly from those of [16]. This manifests itself primarily in our predictions for the expectation value of $\Phi_0$ which we find to be significantly smaller than $\mu$, irrespective of the value of the cut-off. This means that our model does not appear to be a good candidate for gauge mediation, since $\langle \Phi_0 \rangle^2 < F_{\phi_0}$. However, the gravitational corrections to the quarks F-terms open the possibility that they could couple to a mediation sector and generate soft terms. We will return to this in section 7.

## 4.1 Analytic properties of $\mathrm{STr}\mathcal{M}^2$

For a canonical Kähler potential, the quadratic one-loop potential is given by

$$V_{\text{quad.}} = \frac{\mathrm{STr}\mathcal{M}^2 \Lambda^2}{32\pi^2} = \frac{\Lambda^2 e^K}{16\pi^2}(N_f^2 + 2N_f N_c - 1)\left(\sum_i (W_i + X_i W)^2\right) - \frac{\Lambda^2 e^K}{8\pi^2}(N_f^2 + 2N_f N_c)W^2. \tag{4.2}$$

Where fields are taken to be real and $X_i$ runs over all fields. Re-writing this in terms of the tree-level potential gives:

$$V_{\text{quad.}} = \frac{\Lambda^2}{16\pi^2}(N_f^2 + 2N_f N_c - 1)V_{\text{tree}} + \frac{\Lambda^2 e^K}{16\pi^2}((N_f^2 + 2N_f N_c) - 3)W^2 \tag{4.3}$$

and hence

$$V_{\text{tree}} + V_{\text{one-loop}} = \left((N_f^2 + 2N_f N_c - 1)\frac{\Lambda^2}{16\pi^2} + 1\right)V_{\text{tree}} + \frac{\Lambda^2}{16\pi^2}(N_f^2 + 2N_f N_c - 3)e^K W^2 + V_{\text{log}} \tag{4.4}$$

The addition of $V_{\text{quad.}}$ to the potential reinforces the tree-level solution, up to additional, gravitationally suppressed contributions from the final term in eq. (4.3).

On dimensional grounds, the SUSY parts of the Sugra F-terms will provide the dominant contributions to $V_{\text{quad.}}$, except for the moduli fields, which have flat F-terms at the SUSY minimum. This means we expect the minima of $V_{\text{quad.}}$ and $V_{\text{tree}}$, for the non-moduli fields, to coincide at leading order in $\mu$ (assuming the moduli are taken to be $\sim \mu^2$). However, the gravitational corrections given by $W^2$ and by $K_i W$ come in at the same order of magnitude and hence we do not expect that the same $\Phi_0$ will minimise both $V_{\text{quad.}}$ and $V_{\text{tree}}$. It is nonetheless clear from eq. (4.3) that, if $(N_f^2 + 2N_f N_c)\frac{\Lambda^2}{16\pi^2} \sim 1$, the quadratically divergent corrections will be of equal importance to the tree-level.

We may also compute the value of $W_0$ required to cancel the cosmological constant:

$$W_0 = \left(\frac{e^{-K}V_{\text{log}} + \left(N_f' \Lambda'^2 + 1\right)h^2 N_c \mu^2}{3\left(N_f' \Lambda'^2 + 1\right) - (N_f' - 2)\Lambda'^2}\right)^{1/2} \tag{4.5}$$

Where $\Lambda' = \frac{\Lambda}{4\pi}$ and $N_f' = N_f^2 + 2N_f N_c - 1$.

The minima of the logarithmic potential and tree-level potential need not coincide, which is fortunate since the tree-level potential is minimised by Planck scale moduli vevs. Therefore the addition of the logarithmic potential to the tree-level will shift the minimum away from the tree-level, with the size of the shift being determined by the strength of the coupling constants and the relative importance of gravity. Inclusion of the quadratic, one-loop potential will shift the minimum closer to the tree-level result for all fields as discussed in appendix B.

## 4.2 Remarks on methodology

Since computation of $V_{\log}$ requires diagonalisation of $\mathcal{M}^2$ it is significantly more involved than the calculation of the quadratic piece. It is possible to make some analytic progress by using the one-loop moduli masses derived in [1] and tree-level gravity corrections. However, more involved analytic calculations, such as computing the logarithmic piece of eq. (4.1) (with or without gravitational corrections to the masses) or computing corrections to the Kähler potential, as detailed in [15] are extremely challenging since they both rely upon diagonalization of large mass matrices. We can make some approximate analytical statements by observing that the direct gravitational corrections to the logarithmic effective potential for $\Phi_0$ given in [1] are small and overwhealmed by the non-gravitational terms. Then using the global effective potential, derived at the global SUSY minimum, will only introduce small errors if it does not vary too rapidly across field space and the combination of this and the Sugra tree-level potential gives vevs close to the global vevs. To see if this approximation was valid we opted to calculate the effective potential numerically, using the approach described in the following section. One can make progress by observing that, at the minimum, it is possible to compute the series expansion the one-loop potential up to second order in the fields. This leaves enough information to confirm that this is point is both a stationary point and a minimum. Eq. (4.1) can then be computed for pairs of fields at a time, with all others frozen. With the problem broken into several managable parts it is possible to attack it numerically as described in the following section.

However computing the effective Kähler potential is a more difficult task, since one needs to know the series expansion of the matrix of second derivatives of the inverse Kähler potential up to second order in the fields. This is in addition to knowing the Kähler potential up to second order. To calculate the second derivatives of the inverse Kähler potential to second order we would need to calculate the Kähler potential to fourth order, doubling the number of fields we have to consider simultaneously. This alone shows that this approach should be significantly more time consuming.

As a technical point we note that the Coleman-Weinberg contribution to $\Phi_0$'s mass derived in ISS is not valid away from the global ISS minimum. If we only include the one-loop masses derived in [1] and allow all fields to vary, then the numerical solution has negative eigenvalues in the Hessian. This apparent instability can be shown to be an artifact caused by the mismatch between the global minimum (the point at which ISS's masses are correct) and the true minimum (in which they are not). While these effects are small we can re-calculate the one-loop contribution at the minimum of the tree-level plus

one-loop potentials, we can trust our results. Unfortunately, since $\langle \Phi \rangle \neq 0$ and it is no longer a simple matter to determine the Goldstone directions, the calculation of the masses must be done for all fields and becomes significantly more involved.[5] As a result of this, we discovered non-trivial contributions to not only $\Phi_0$'s potential, but $\phi_0$ and $\Phi_1$'s.

Our approach was to start at a point close to the global ISS minimum, compute the one-loop potential to second order in the fields at this point and then minimise the tree-level + one-loop potential.[6] This new point was then used to re-compute the one-loop potential, allowing us to minimise yet again. This process was repeated until the solution converged and the eigenvalues of the matrix of second derivatives of the full potential were consistent with the expected number of Goldstones (which would not be the case if the effective potential were computed away from the minimum).

Regrettably, this is not sufficient to make the computation tractable for large numbers of flavours, as the computation time grows rapidly with the number of flavours and $N_f = 4$, $N_c = 3$ is already takes prohibitively long. However, for $N_f = 3$, $N_c = 2$ the one-loop potential can be computed in a reasonable amount of time.

## 5 Non-perturbative contributions

To estimate the value of the cut-off for which the non-perturbative piece (2.4) dominates, and destabilises the potential, we calculate the second derivative of the non-perturbative correction to the potential, evaluated at the minimum of the tree-level + one-loop effective potential. If $\frac{\partial^2 V_{\text{non-pert.}}}{\partial \Phi^2} > \frac{\partial^2 \left( V_{\text{tree}} + V_{\text{one-loop}} \right)}{\partial \Phi^2}$, then the non-perturbative piece will likely dominate and the fields will roll to the supersymmetric minimum. This allows us to put rough lower bounds on the cut-off, such that the meta-stable solution is stable. This is in contrast to the global case in which the non-perturbative effects vanish in the tree-level + one-loop minimum. Considering, for simplicity's sake, $N_f = N_c + 1$:

$$\frac{\partial^2 V_{\text{non-pert.}}}{\partial \Phi_1 \partial (\Phi_0)_i} \sim \sum_j^{N_f - 1} 2h^{N_f/N} \Lambda_m^{-(N_f - 3)} \left( \prod_{k \neq j, i}^{N_f - 1} (\Phi_0)_i \right) (-h\mu^2) \tag{5.1}$$

where all higher powers of $\Phi$ have been discarded.

Close to the global SUSY minimum the dominant contribution to the effective potential is the globally supersymmetric effective potential derived in [1]. This means that

$$\frac{\partial^2 V_{\log}}{\partial \Phi_{ii}^2} \sim \frac{\log(4) - 1}{4\pi^2} h^4 \mu^2 \tag{5.2}$$

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[5]This is in contrast to global ISS, or indeed ISS with no additional physics, because in our case $\langle \Phi \rangle \neq 0$. When $\langle \Phi \rangle = 0$ the Goldstone bosons are solely linear combinations of the magnetic quarks, but when $\langle \Phi \rangle \neq 0$ it contributes to the breaking of $\mathrm{SU}(N_f)$ and hence the Goldstone bosons. This means that any contributions to the Goldstone bosons' potential that violate the original $\mathrm{SU}(N_f)$ symmetry give (spurious) masses to the Goldstones. Hence one should compute the full potential, at least to 2nd order in the fields, to obtain reliable results.

[6]This avoids the need to diagonalise the mass matrix analytically. Which would be challenging because it is both large and has numerous independent variables. Since we only need the local properties of the potential we only need calculate the its numerical values after small variations in the fields. If one wanted to know the values of the potential for all values of the fields, it would be necessary to diagonalise analytically.

and hence if $\sum_j 2h^{\frac{N_f+N}{N}} \Lambda_m^{-(N_f-3)} \left( \prod_{k \neq j,i}^{N_f-1} (\Phi_0)_i \right) > \frac{\log(4)-1}{4\pi^2} h^4$ it is clear that the non-perturbative piece will dominate and the fields will evolve into the SUSY minimum. However, we can see that, even if the cut-off has the same order of magnitude as $\mu$, the non-perturbative potential alone will not have a significant effect. It is suppressed by the small expectation values of $\Phi_0$, given by the perturbative potential. If $\langle \Phi \rangle \gg \mu$ the non-perturbative potential can dominate, but this is far from the case here.

We also note that, while there are non-perturbative contributions to the one-loop effective potential, these effects are generically small. They will only need close consideration when $\langle \Phi \rangle \gg \mu$ and even in this case, the non-perturbative contributions to the tree-level will be more important, except at singular points where the perturbative expansion breaks down.

Finally, the complete calculation with a fully realistic number of flavours is numerically intractable, and we are only able to compute everything for the case where $N_f = 3$ and $N_c = 2$. While this case does not correspond to a dualized theory, it does capture the important low energy phenomena: the rank condition still holds and SUSY is still broken spontaneously. However, the non-perturbative contributions are qualitatively different. Firstly the coefficient is automatically 1, irrespective of the size of the cut-off, and the determinant piece is larger by roughly $h^2\mu^{-2}$, since it contains one fewer power of $\Phi_{ij}$. This means that the non-perturbative piece can come to dominate, even though we know that it would be negligible in the $N_f = 4$, $N_c = 3$ case. As a result, we were forced to introduce a constant, $\Lambda_{\text{non-pert.}}$, multiplying the non-perturbative piece and find the largest stable value, before we could be certain that the non-perturbative effects were under control. More specifically

$$W_{non-pert} \to W_{non-pert} = \Lambda_{\text{non-pert.}} N h^{N_f/N} \left( \Lambda_m^{-(N_f-3N)} \det(\Phi) \right)^{1/N} \qquad (5.3)$$

in what follows.

## 6 Supersymmetry breakdown

### 6.1 Numerical results

We now present the numerical analysis of our model. In the following we confirm that the metastable minimum exists in the presence of gravity, show where the numerical results diverge from the analytical approximations and explain why these deviations are larger than dimensional analysis suggests. Where possible we compare our results with those presented previously, demonstrating that they can be recovered if the same assumptions about the one-loop potential are made (the assumptions used to derive the analytical approximations), but that relaxing these assumptions introduces the shift just discussed.

Our main observation in this section is that one must be careful about estimating the error introduced by neglecting gravitational corrections to the one-loop effective potential. As far as the logarithmic one-loop potential is concerned, it is necessary to compute it in full, though the gravitational corrections can safely be ignored.[7] The quadratic one-loop

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[7]We included them in our numerical analysis for completeness, but the corrections proved to be small.

| $N_f$ | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
|---|---|---|---|---|---|---|---|
| $N$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\mu$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ |
| $\Lambda_{\text{non-pert.}}$ | 0 | $2.00\ 10^{-6}$ | $2.00\ 10^{-4}$ | $2.00\ 10^{-2}$ | $4.00\ 10^{-2}$ | $4.40\ 10^{-2}$ | $4.60\ 10^{-2}$ |
| $m_{3/2}$ | $1.52\ 10^{-14}$ | $1.52\ 10^{-14}$ | $1.52\ 10^{-14}$ | $1.52\ 10^{-14}$ | $1.52\ 10^{-14}$ | $1.52\ 10^{-14}$ | $1.52\ 10^{-14}$ |
| $\langle\phi_0\rangle$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ |
| $\langle\Phi_0\rangle$ | $-6.33\ 10^{-12}$ | $-6.33\ 10^{-12}$ | $-6.33\ 10^{-12}$ | $-7.95\ 10^{-12}$ | $-1.57\ 10^{-11}$ | $-2.10\ 10^{-11}$ | $-2.56\ 10^{-11}$ |
| $\langle\Phi_1\rangle$ | $-1.21\ 10^{-13}$ | $-1.21\ 10^{-13}$ | $-1.23\ 10^{-13}$ | $-3.71\ 10^{-13}$ | $-1.16\ 10^{-12}$ | $-1.66\ 10^{-12}$ | $-2.08\ 10^{-12}$ |
| $F_{\phi_0}/\mu^2$ | $-7.43\ 10^{-7}$ | $-7.43\ 10^{-7}$ | $-7.56\ 10^{-7}$ | $-2.50\ 10^{-6}$ | $-8.03\ 10^{-6}$ | $-1.15\ 10^{-5}$ | $-1.45\ 10^{-5}$ |
| $F_{\Phi_1}/\mu^2$ | $-1.07\ 10^{-2}$ | $-1.07\ 10^{-2}$ | $-1.07\ 10^{-2}$ | $-1.07\ 10^{-2}$ | $-1.07\ 10^{-2}$ | $-1.07\ 10^{-2}$ | $-1.07\ 10^{-2}$ |

**Table 1**. Solutions for V=0, with the non-perturbative piece included. The dependence on the non-perturbative contribution is shown to be very small, but as $\Lambda_{\text{non-pert.}}$ exceeds $2\ 10^{-2}$ it rapidly comes to dominate. The value of $m_{3/2}$ is identified with $W_0$, up to small corrections, supressed by additional powers of $\mu$.

potential must be included. While the quadratic one-loop potential is zero in the absence of gravity, the gravitational corrections generate a potential similar to the gravitational corrections to the tree-level, but controlled by an overall factor of $\frac{\Lambda^2}{32\pi^2}$ - as we show in appendix B. These potentials, combined with the tree-level, must be computed in order to obtain a reliable leading order estimate in general, with the quadratic potential being of particular interest if the number of fields is large and the cut-off close to the Planck scale.

In table 1 the row $\Lambda_{\text{non-pert.}}$ is the coefficient of the non-perturbative piece, introduced to compensate for the missing powers of $\Phi_{ij}$ that would be present in the determinant for a realistic number of flavours. Note, the F-term for $\Phi_0$ is not included since it is only shifted by corrections of order $\mu^3$, so $F_{\Phi_0} = h\mu^2 + \mathcal{O}(\mu^3)$. For physically reasonable values of $\mu$ this effect is negligible. Also, $\Lambda = 10^{-2}$, (i.e. the string/GUT scale) throughout tables 1 and 3. The reason is, this value of $\Lambda$ is sufficiently small and ensures that the quadratically divergent loop correction to the potential only creates a small shift from the tree-level result, leaving us with the logarithmic piece whose sensitivity to the value of the cut-off is very weak.

Since the superpotential can be written $W = W_{ISS} + hW_0$, and $\langle W_{ISS}\rangle = 0$ in the global limit, we find that $m_{3/2} = e^K h W_0 + \mathcal{O}(\mu^3) \simeq hW_0$. Hence we only include $m_{3/2}$ in the tables. Also, in all the tables $h$ is taken to be 1. Our main result, concerning the soft mass spectrum, is independent of the precise value of $h$, though we retain full theoretical control in a surprisingly small range. If $h \ll 10^{-1}$ the $\Phi_0$ modulus runs off to the Planck scale as the gravitational effects overwhelm the one-loop contributions (since $\mu$ must go like $\mu \to \frac{\mu}{h}$, as $h$ varies from 1, and hence gravitational effects increase in relevance as $h$ shrinks). However, if $h \gg 1$ the theory becomes strongly coupled.

Our numerical studies show that the non-perturbative piece is under control, that there is only a very mild, logarithmic cut-off dependence, when supergravity corrections are accounted for, and that the main features of the model are independent of $\mu$, assuming $\mu \ll M_P$. These results can be seen in, respectively, tables 1, 2 and 3.

| $N_f$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
|---|---|---|---|---|---|---|---|
| $N$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\mu$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ |
| $\Lambda$ | 1 | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ | $2\ 10^{-7}$ |
| $m_{3/2}$ | $1.04\ 10^{-14}$ | $1.06\ 10^{-14}$ | $1.08\ 10^{-14}$ | $1.09\ 10^{-14}$ | $1.11\ 10^{-14}$ | $1.13\ 10^{-14}$ | $1.16\ 10^{-14}$ |
| $\langle\phi_0\rangle$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ | $1.41\ 10^{-7}$ |
| $\langle\Phi_0\rangle$ | $-4.29\ 10^{-12}$ | $-4.36\ 10^{-12}$ | $-4.43\ 10^{-12}$ | $-4.50\ 10^{-12}$ | $-4.57\ 10^{-12}$ | $-4.64\ 10^{-12}$ | $-4.75\ 10^{-12}$ |
| $\langle\Phi_1\rangle$ | $-4.78\ 10^{-14}$ | $-4.44\ 10^{-14}$ | $-4.16\ 10^{-14}$ | $-3.91\ 10^{-14}$ | $-3.70\ 10^{-14}$ | $-3.51\ 10^{-14}$ | $-3.25\ 10^{-14}$ |
| $F_{\phi_0}/\mu^2$ | $-2.64\ 10^{-7}$ | $-2.39\ 10^{-7}$ | $-2.17\ 10^{-7}$ | $-1.99\ 10^{-7}$ | $-1.83\ 10^{-7}$ | $-1.68\ 10^{-7}$ | $-1.47\ 10^{-7}$ |
| $F_{\Phi_1}/\mu^2$ | $-5.66\ 10^{-3}$ | $-5.46\ 10^{-3}$ | $-5.27\ 10^{-3}$ | $-5.09\ 10^{-3}$ | $-4.93\ 10^{-3}$ | $-4.77\ 10^{-3}$ | $-4.53\ 10^{-3}$ |

**Table 2**. Solutions for V=0 where the non-perturbative and one-loop quadratic pieces have been neglected. These data show the logarithmic dependence of $\Phi_0$ on the cut-off.

From the data in table 2 we can see that for $\mu = \sqrt{2}\ 10^{-7}$, $\langle\Phi_0\rangle = -4.295\ 10^{-12} + 6.886\ 10^{-14}\mathrm{Log}_{10}(\Lambda)$. Table 3 shows that there are no significant changes as one varies $\mu$. In table 1 $\Lambda_{\text{non-pert.}}$ is varied and the non-perturbative term becomes relevant for $\Lambda_{\text{non-pert.}} > 4\ 10^{-2}$. Since we expect the non-perturbative term to be suppressed by an extra power of $\Phi$ when $N_f = 4$, corresponding to $\Lambda_{\text{non-pert.}} \sim \Phi \sim \mu^2 = 2\ 10^{-14}$ it is clearly well under control.

## 6.2 Analytic approximations

In addition we can compare our numerical results to analytical approximations. Because of the expected smallness of the fields it is sufficient to take the leading order of $\mu$ when searching for the minimum. The gravitational corrections to the logarithmic one loop potential appear with higher powers of $\mu$ than the SUSY effective potential and can be neglected in the analytical approximation, while the quadratic corrections are both relevant and readily calculable. The solution to $\frac{\partial V}{\partial \Phi_0} = 0$, derived in appendix B, under the assumption[8] that $V_{\log}$ is described by eq. (B.7), is given by:

$$\langle\Phi_0\rangle = \frac{16\pi^2 W_0(1 + \frac{\Lambda'^2}{2}(N_f' + 2))}{h^2(\log(4) - 1)} + \mathcal{O}(\mu^4) \tag{6.1}$$

Comparing this with the numerical results, obtained using the full one-loop potential, shows that they disagree at the percent level. For example, it gives $\langle\Phi_0\rangle = -4.25\ 10^{-12}$, with $N_f = 2$, $\mu = \sqrt{2}\ 10^{-7}$, $\Lambda = 1$ and $V_{\text{quad.}}$ neglected (i.e. $\Lambda' = 0$), to be compared with table 2. Since the difference is far greater than the error we expected to be introduced by ignoring gravitational corrections to the one-loop potential, we conclude that the assumption that the one-loop potential is given by eq. (B.7) is not justified in the presence of gravity. This deviation is a consequence of the changes in the effective potential for $\Phi_0$ caused by moving from the global SUSY minimum to the Sugra minimum, in $\Phi_1$-$\phi_0$

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[8]That the one-loop potential is given entirely by the globally supersymmetric one-loop potential derived in [1].

| $N_f$ | 2 | 2 | 2 | 2 |
|---|---|---|---|---|
| $N$ | 1 | 1 | 1 | 1 |
| $\mu$ | $10^{-7}$ | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ |
| $m_{3/2}$ | $5.36\ 10^{-15}$ | $5.45\ 10^{-13}$ | $5.54\ 10^{-11}$ | $5.63\ 10^{-9}$ |
| $\langle\phi_0\rangle$ | $9.97\ 10^{-8}$ | $9.97\ 10^{-7}$ | $9.98\ 10^{-6}$ | $9.98\ 10^{-5}$ |
| $\langle\Phi_0\rangle$ | $-2.21\ 10^{-12}$ | $-2.25\ 10^{-10}$ | $-2.28\ 10^{-8}$ | $-2.32\ 10^{-6}$ |
| $\langle\Phi_1\rangle$ | $-2.10\ 10^{-14}$ | $-1.97\ 10^{-12}$ | $-1.86\ 10^{-10}$ | $-1.77\ 10^{-8}$ |
| $F_{\phi_0}/\mu^2$ | $-1.56\ 10^{-7}$ | $-1.42\ 10^{-6}$ | $-1.31\ 10^{-5}$ | $-1.20\ 10^{-4}$ |
| $F_{\Phi_1}/\mu^2$ | $-5.30\ 10^{-3}$ | $-5.12\ 10^{-3}$ | $-4.95\ 10^{-3}$ | $-4.79\ 10^{-3}$ |

**Table 3**. Solutions for V=0 where the non-perturbative and one-loop quadratic pieces have been neglected. Here, the $\mu$ dependence is shown.

space. If we artificially shift all fields except $\Phi_0$ to their global SUSY vevs then the Sugra corrected logarithmic effective potential for $\Phi_0$ tends to the SUSY effective potential. The gravitational corrections to $V_{\log}$ at the global SUSY minimum are negligible, as expected. It is also interesting to see that at this point in $\Phi_1$-$\phi_0$ space the one-loop correction to the potential contains a term linear in $\Phi_0$ and hence is minimised by $\langle\Phi_0\rangle \neq 0$.

We also notice similar behaviour when $V_{\text{quad.}}$ is included. For $N_f = 2$, $\mu = \sqrt{2}\ 10^{-7}$, $\Lambda = 1$ eq. (6.1) predicts $\langle\Phi_0\rangle = -4.37\ 10^{-12}$, but the minimum appears at $\langle\Phi_0\rangle = -4.46\ 10^{-12}$.

As noted in appendix A, when $W_0 = 0$, the expectation values of the quarks are determined by the global minimisation conditions modified by the expectation value of the tree-level potential. Cancelling the cosmological constant at tree-level, via $W_0$, recovers the global result, up to small corrections induced by $W_0$. When the loop corrections are removed, $\Phi \to 0$ and $N_c = 1$ we find

$$\langle\phi_0\rangle^2 = \frac{1}{2}\left(-1 + 2\mu^2 - 2W_0^2 + \sqrt{1 - 4N_c\mu^4 + 12W_0^2 - 8\mu^2W_0^2 + 4W_0^4}\right) \tag{6.2}$$

$$= \mu^2 - N_c\mu^4 + 2W_0^2 - 2\mu^2W_0^2 + \mathcal{O}(\mu^8) \tag{6.3}$$

hence, to good approximation, $\langle\phi_0\rangle^2 = \mu^2$ for $\mu = \sqrt{2}\ 10^{-7}$, in agreement with the leading order calculation in appendix B. At $\mu^4$ order this result depends on the cancellation of the cosmological constant, requiring the equality of the $F$ term contribution and $-3W^2$. Hence, the one loop potential would shift $\langle\phi_0\rangle^2$ by $\lesssim \mu^4$, even if it were independent of $\phi_0$ (as has tacitly been assumed in [3, 9, 22] and [25]) and merely contributed to the cosmological constant. Moreover, the one-loop potential proves to have a non-trivial dependence on $\phi_0$ and, for $h = 1$, introduces a correction at the level of $\frac{\mu}{1000}$. It should be stressed that this effect remains when the one-loop potential is purely supersymmetric, as long as gravity is switched on in the tree level potential. For $\Lambda = 1$, $N_f = 2$ and $N_c = 1$ we find $\langle\phi_0\rangle = 1.41042\ 10^{-7}$ if $V_{\text{quad.}}$ is present and $\langle\phi_0\rangle = 1.41020\ 10^{-7}$ if it is not. In contrast, these two results would be indistinguishable at this level of precision had only the logarithmic corrections to $\Phi_0$'s potential been accounted for.

Similarly to $\Phi_0$, $\Phi_1$'s logarithimic one-loop potential depends on the moduli expectation values and, unlike the quarks, does not tend to the tree-level result as $h \to 0$ (and $V_{\text{quad.}}$ is neglected). For $N_f = 2$, $\mu = 10^{-7}$, $\Lambda = 1$ and with $V_{\text{quad.}}$ neglected we obtain $\langle \Phi_1 \rangle = -4.78 \ 10^{-14}$ which differs by a factor of 5 compared to the leading order tree-level result of $\langle \Phi_1 \rangle = -W_0 = -1.04 \ 10^{-14}$ derived in appendix B. The results with $V_{\text{quad.}}$ included are closer as the tree-level + quadratic potential also gives $\langle \Phi_1 \rangle = -W_0 = -1.04 \ 10^{-14}$, and the full potential gives $\langle \Phi_1 \rangle = -4.65 \ 10^{-14}$.

## 7 Soft masses

It is well known that supergravity theories automatically include the gravitational mediation mechanism and soft-terms will be generated. The typical scale for these soft masses is $m_{3/2}$ with deviations being generated by non-trivial Kähler potentials. Also, gaugino masses can be zero at tree-level, if the gauge kinetic function preserves supersymmetry. Since we know the gravitational contributions must be present we now analyse the relative importance of gauge and anomaly mediation and sketch the features of the spectrum.

Since the addition of a constant to the superpotential implies that $\langle \Phi \rangle \neq 0$, in supergravity, we investigated the possibility that the R-symmetry violating, non-perturbative piece could give masses to the gauginos. The determinant piece (2.4) can have a non-trivial contribution to the ISS fields' masses and, if we employed a direct mediation mechanism analogous to, for example, [17], the R-symmetry breaking could in principle be transfered to the MSSM sector. However, as we saw in table 1, the square of the scalar vev of $\Phi$ is less than the F-term. Hence, $\Phi$ cannot be the 'X' field[9] that couples to the messengers since, if $\langle \Phi \rangle = M + F\theta^2$ then $M^2 < F$, and, if $\langle \Phi \rangle$ is the sole contributor to the messenger masses, then they will be tachyonic (as shown in [18]).

However, the gravitational effects induce new F-terms that are not present in global SUSY. Being generated gravitationally, they are always smaller than the F-terms of $\Phi_0$, assuming the gravitational effects are under control.[10] Since the magnetic squarks' vevs are $\sim \mu$ and their F-terms $\sim h\mu^3$ they automatically satisfy the $M^2 > F$ condition, if $h \leq 1$. An example messenger sector would have the symmetry group $SU(N) \times SU(N_f) \times SU(5) \times U(1)_R$ and the fields would transform as follows: $\phi_0 : (N, N_f, 1, 0)$, $f : (\bar{N}, 1, 5, 1)$, $f' : (1, \bar{N}_f, \bar{5}, 1)$.

The crucial observation is that two SUSY breaking scales are generated automatically, if $V = 0$ is required. Hence the standard argument for the dominance of gauge mediation, namely that $\frac{F}{M} \gg \frac{F}{M_P}$, does not necessarily apply. Instead we have $F_{\Phi_0} \gg F_{\phi_0}$ and $\frac{F_{\phi_0}}{\langle \phi_0 \rangle} \sim \frac{F_{\Phi_0}}{M_P}$. While it is true that $\frac{F_{\Phi_0}}{\langle \Phi_0 \rangle} \gg \frac{F_{\Phi_0}}{M_P}$, $\Phi_0$ cannot be allowed to couple to the messengers because $F_{\Phi_0} > \langle \Phi_0 \rangle^2$.

The naive expression for the gaugino masses is given by:

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[9]We have in mind an operator $W \ni Xf\bar{f}$ where $f$ and $\bar{f}$ are messenger fields charged under the visible sector gauge groups.

[10]Since the gravitational effects come in with an additional power of $\mu$ compared with the global SUSY terms and the new F-terms are $\sim h\mu^3$.

$$m_\lambda \sim \frac{\alpha}{4\pi} \frac{F_{\phi_0}}{\langle \phi_0 \rangle} \sim \frac{\alpha}{4\pi} \frac{h \langle \phi_0 \rangle W_0}{\langle \phi_0 \rangle} \sim \frac{\alpha}{4\pi} h\mu^2 \tag{7.1}$$

and the soft scalar mass squareds are approximately:

$$m^2 = m_\lambda^2. \tag{7.2}$$

In addition we can estimate the soft mass contributions from anomaly mediation:

$$M_a = F_{\text{Anom.}} \beta_{g_a}/g_a \sim \frac{\alpha}{4\pi} F_{\text{Anom.}} \tag{7.3}$$

$$(m^2)^i_j = F^2_{\text{Anom.}} \frac{\partial \gamma^i_j}{\partial t} \sim \frac{\alpha^2}{16\pi^2} F^2_{\text{Anom.}}. \tag{7.4}$$

Since $F_{\text{Anom.}}$ can at most be $F_{\Phi_0}$ (without postulating another source of SUSY breaking) it follows that these contributions are of the same order of magnitude as those given by gauge mediation.

This allows us to estimate the size of the gravitino mass, based on the requirement that the gaugino masses be in the vicinity of a TeV and that the cosmological constant be tuned to zero. These requirements imply that $\frac{\alpha}{4\pi} h\mu^2 \sim 1$ TeV $\sim 10^{-16}$ and thus $h\mu^2 \sim \frac{4\pi}{\alpha} 10^{-16}$. Taking $\alpha = \frac{1}{26}$ [19] we find $h\mu^2 \sim 3 \ 10^{-14}$, up to order one factors, and we see that $m_{3/2} \simeq e^K h W_0 \simeq \left( \frac{h^2 N_c \mu^4}{3} \right)^{1/2}$ and hence the gravitational contribution to the soft scalar masses is automatically two orders of magnitude larger than the other contributions.

As mentioned earlier, the gravitational contribution to the gaugino masses is not necessarily order $m_{3/2}$. For example in string theories one can find that the gauge kinetic functions, whose expectation values specify the gauge coupling constants, have a tree level dependence on closed string moduli [20].[11] Since pseudo-moduli are evidently not string moduli, the tree-level gauge kinetic function does not have to depend on them. Hence, if the string moduli (or any fields that appear in the gauge kinetic function) do not contribute strongly to SUSY breaking[12] then the gaugino masses will receive a negligible tree-level contribution from gravity.

In summary, the models discussed here may naturally generate, via the gauge mediation channel, a spectrum in which the gaugino masses are loop suppressed with respect to the soft scalar masses. Since the gaugino masses receive unknown quantum corrections, we cannot predict the precise spectrum, which depends on details of the complete model, though

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[11]Unlike the pseudo-moduli discussed here, which only have flat directions at special points in field space, these moduli have no potentials classically. Non-perturbative corrections are required to give string moduli potentials, whereas pseudo-moduli have potentials at tree-level and the flat directions are removed at the one-loop level.

[12]As we expect in certain racetrack models. For example the O'KKLT class of models [21] have finely-tuned moduli sectors in which, when considered alone, have SUSY-preserving, De-Sitter minima. The addition of an O'Raifeartaigh sector [10] (the possibility that this might be ISS was considered in [22]) spontaneously breaks SUSY and lifts the vacuum energy. Since the two sectors are decoupled at the global level the moduli F-terms can be significantly smaller than the other fields'.

we do expect to see a split in the spectrum of soft masses. This spectrum has much in common with the one presented in the early work on anomaly mediation. See, for example, [23].

In addition to this, one can consider modifying the model. For example, there are examples [17, 24–26] in which operators are added to the superpotential allowing the pseudo-modulus vev to grow out to around $\mu$. While this is clearly an interesting effect, the dual theory responsible for generating this operator is as yet unknown.

## 8   Conclusions

In summary, ISS is more stable with supergravity corrections than without, assuming that $W_0 = 0$. The picture changes somewhat when $W_0 \neq 0$, but it was shown in section 4 that the supergravity effects are under good theoretical control.

It was then demonstrated, in section 5 that, while the non-perturbative piece is non-zero, it is necessarily sub-dominant for small values of $\Phi$. Hence this term can be neglected when supergravity effects are subdominant to the one-loop effects. Numerical analysis confirmed this.

We also showed that the quadratically divergent, one-loop potential's effects are small and under control if $\Lambda' N'_f \ll 1$. We have constructed our theory such that supersymmetry breaking is only generated by the ISS sector, to highlight the distinctive features of this model. An example of a possible a high energy model was considered in [7], but we did not attempt this kind of construction in this paper. Also, it is worth noting that it is consistent to make use of the supergravity corrected effective potential, even though our cut-off can be taken to be many orders of magnitude below. This is because the non-renormalisable operators present in Sugra are not generated by the integration out of gravitational fields, and are instead required to be present by supersymmetry itself.

The explicit R-breaking introduced by the presence of a constant term in the superpotential allows the generation of non-zero gaugino masses, through gravitationally suppressed interactions (as one would expect, since global SUSY is blind to the presence of the constant).

We showed, in section 7, that in the absence of additional supersymmetric contributions to messenger masses, the gauge mediation is only possible if the magnetic quarks couple to the messenger fields. All other ISS fields have overly small scalar vevs and give rise to an unstable messenger sector. The direct consequence of which is that the gravitino mass is quite large. This is the case because it is not set by the quarks F-terms, but the much larger meson F-terms, through the requirement that the cosmological constant should vanish in the minimum of the effective potential. This results in a direct connection between R-symmetry breaking and gaugino masses, with eq. (7.1) showing that they are proportional to one another, with the coefficient being determined by the details of the gauge group.

### Acknowledgments

## A  Origins of mass terms

For simplicity we neglect phases in the following analysis. Hence all symmetries effectively go from $\mathrm{U}(N) \to O(N)$ and the counting of degrees of freedom reflects this.

In this case, the tree-level potential, when embedded into supergravity, has a metastable minimum. The position of the minimum is given by the global result, but with small corrections to the expectation value of the magnetic quarks $|\phi_0|^2 = |\tilde{\phi}_0|^2 = \mu^2 - 1/2 \pm \frac{1}{2}(1 - 4N_c\mu^4)^{1/2} \sim \mu^2 - N_c\mu^4$. This deviation from the global limit only comes from the overall factor of $E^K$ multiplying the potential.[13]

The spectrum contains $2N_f N$ magnetic quarks of which $\frac{1}{2}(2N_f N - N^2 - N)$ are massless Goldstone bosons and the remainder have masses of order $\mu$, $2N_f N - N^2$ mesons with mass of order $\mu$, $(N_f - N)^2 - 1$ mesons with mass of order $\mu^2$ and one massless pseudo-moduli meson.

The origins of these masses are as follows. The quarks get their masses from their expectation values, with $\phi_0$ giving mass to $\tilde{\phi}_0$ and vice versa, the off-diagonal elements of $\Phi_0$ obtain masses solely from the second derivative of $e^K$ (which contributes equally to all fields), namely $2V$, while the diagonal elements get more complicated contributions. The remaining elements of $\Phi$ get the same masses as in global SUSY, but with small corrections from the Sugra contributions. The end result of this is that the massive fields retain essentially same masses as in global SUSY and all but one of the pseudo-moduli (which remains zero) obtain masses of the order of the cosmological constant: $2N_c\mu^4$. This demonstrates that supergravity serves to increase the stability of the ISS minimum, as it reinforces the stabilising effects coming from the one loop potential.

## B  Analytical solutions

Here we present the approximate analytical expressions for the derivatives of the tree-level and quadratic, one-loop potentials. Since both potentials can be written in terms of $V_F$ and $V_W$ we compute the derivatives of these functions, taking the fields to be real.

$$\frac{\partial V_F}{\partial \Phi_1} = 2h^2\left(\tilde{\phi}_0(\tilde{\phi}_0\Phi_1 + \phi_0 W_0) + \phi_0(\phi_0\Phi_1 + \tilde{\phi}_0 W_0) + W_0(\phi_0\tilde{\phi}_0 - \mu^2)\right) + \mathcal{O}(\mu^6) \sim h^2\mu^4 \tag{B.1}$$

$$\frac{\partial V_F}{\partial \Phi_0} = -2h^2 W_0\mu^2 + \mathcal{O}(\mu^6) \sim h^2\mu^4 \tag{B.2}$$

$$\frac{\partial V_F}{\partial \phi_0} = 2h^2\tilde{\phi}_0(\phi_0\tilde{\phi}_0 - \mu^2) + \mathcal{O}(\mu^5) \sim h^2\mu^3 \tag{B.3}$$

The derivatives of $V_W = -3e^K W^2$ are easily computed and given below, again taking

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[13]In the limit where $\Phi \to 0$ and $W \to 0$ all contributions aside from the overall exponential vanish.

the fields to be real

$$\frac{\partial V_W}{\partial \Phi_1} = -6h^2 W_0(\tilde{\phi}_0 \phi_0 - \mu^2) + \mathcal{O}(\mu^6) \sim h^2\mu^4 \tag{B.4}$$

$$\frac{\partial V_W}{\partial \Phi_0} = 6h^2 W_0 \mu^2 + \mathcal{O}(\mu^6) \sim h^2\mu^4 \tag{B.5}$$

$$\frac{\partial V_W}{\partial \phi_0} = -3h^2 W_0 \tilde{\phi}_0 \Phi_0 - 3h^2 \phi_0 W_0^2 + \mathcal{O}(\mu^7) \sim h^2\mu^5 \tag{B.6}$$

Hence we see that while both $\Phi_0$ and $\Phi_1$ depend directly on $W$, $\phi_0$ does not. This implies that it will remain at the global SUSY minimum, up to corrections induced by the logarithmic piece.

We can make use of the global expression for the logarithmic contribution to $\Phi_0$'s potential,

$$V_{\log} = \frac{h^4 \mu^2 (\log(4) - 1)}{8\pi^2} \text{Tr}(\Phi_0)^2 \tag{B.7}$$

and estimate $\Phi_0$'s expectation value, without the quadratic contribution,

$$\langle \Phi_0 \rangle = \frac{16\pi^2 W_0}{h^2(\log(4) - 1)} + \mathcal{O}(\mu^4), \tag{B.8}$$

This is one of the results previously obtained in, for example [9], under the assumption that the one-loop potential presented in [1] was valid away from the minimum in which it was derived and that the potential for $\Phi_1$ and $\phi_0$ is flat. However, we show numerically that neither of these assumptions are valid, given $h \sim 1$, and hence the one-loop potential plays a more significant role than has been previously discussed.

While we do not have estimates for the logarithmic contributions for $\Phi_1$, $\phi_0$ and $\tilde{\phi}_0$, we can obtain the tree-level expectation values,

$$\langle \Phi_1 \rangle = \frac{-\mu^2 W_0}{\phi_0 \tilde{\phi}_0} + \mathcal{O}(\mu^4) \tag{B.9}$$

and

$$\langle \phi_0 \rangle = \left\langle \tilde{\phi}_0 \right\rangle = \mu + \mathcal{O}(\mu^3) \tag{B.10}$$

and from this point forward we take $\phi_0 = \tilde{\phi}_0$. Eq. (B.10) and eq. (B.9) give

$$\langle \Phi_1 \rangle = -W_0 + \mathcal{O}(\mu^4). \tag{B.11}$$

In addition to this, we can compute the effects of the quadratic, one-loop potential. Taking eq. (4.4) and ignoring the log piece, we obtain

$$V_{\text{tree}} + V_{\text{quad.}} = \left(N'_f \Lambda'^2 + 1\right) V_F - (3 + 2\Lambda'^2(N'_f + 1))e^K W^2. \tag{B.12}$$

First we observe that $\langle \phi_0 \rangle$ will be unchanged as the quadratic contribution, at leading order, simply increases the coefficient of $V_F$ from 1 to $1 + \Lambda'^2 N_f'$.

$$\frac{\partial (V_{\text{tree}} + V_{\text{quad.}})}{\partial \Phi_1} = 4h^2(N_f'\Lambda'^2 + 1)\phi_0^2\Phi_1 + 2h^2\phi_0^2 W_0(N_f' - 2)\Lambda'^2 + 2h^2\mu^2 W_0(2 + 2\Lambda'^2 + N_f'\Lambda'^2) \tag{B.13}$$

$$\frac{\partial (V_{\text{tree}} + V_{\text{quad.}})}{\partial \Phi_0} = 4h^2 W_0\mu^2\left(1 + \frac{\Lambda'^2}{2}(N_f' + 2)\right) + \mathcal{O}(\mu^6) \sim h^2\mu^4 \tag{B.14}$$

and hence

$$\langle \Phi_0 \rangle = \frac{16\pi^2 W_0(1 + \frac{\Lambda'^2}{2}(N_f' + 2))}{h^2(\log(4) - 1)} + \mathcal{O}(\mu^4) \tag{B.15}$$

and

$$\langle \Phi_1 \rangle = \frac{-W_0(\phi_0^2\Lambda'^2(N_f' - 2) + \mu^2(2 + 2\Lambda'^2 + N_f'\Lambda'^2)}{2\phi_0^2(N_f'\Lambda'^2 + 1)} + \mathcal{O}(\mu^4) = -W_0 + \mathcal{O}(\mu^4). \tag{B.16}$$

It is interesting to note that the quadratic potential reinforces the tree-level solution, since, when $\phi_0 \to \mu$, they have the same form. We confirm this in section 6.1.

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